APPENDIX

Optical Electronics in Modern Communications

Fifth Edition

Amnon Yariv
California Institute of Technology

New York Oxford
Oxford University Press
1997

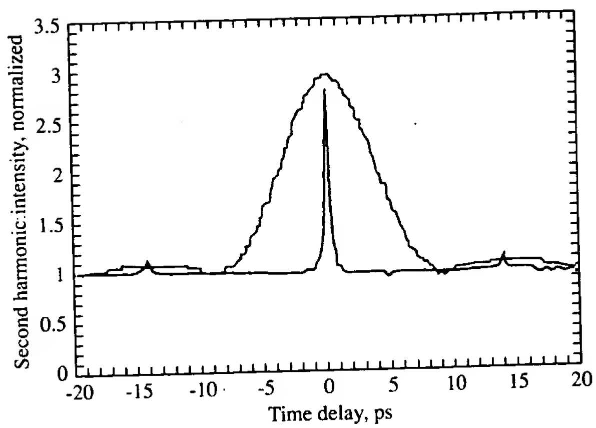


Figure 6-26 Intensity autocorrelation of uncompressed and compressed pulses. The compression ratio is $\sim A = 16$. (After Reference [43]).

Caver

6.9 GIANT PULSE (Q-switched) LASERS (19)

The technique "Q-switching" is used to obtain intense and short bursts of oscillation from lasers; see References [16–18]. The quality factor Q of the optical resonator is degraded (lowered) by some means during the pumping so that the gain (that is, inversion $N_2 - N_1$) can build up to a very high value without oscillation. (The spoiling of the Q raises the threshold inversion to a value higher than that obtained by pumping.) When the inversion reaches its peak, the Q is restored abruptly to its (ordinary) high value. The gain (per pass) in the laser medium is now well above threshold. This causes an extremely rapid buildup of the oscillation and a simultaneous exhaustion of the inversion by stimulated $2 \rightarrow 1$ transitions. This process converts most of the energy that was stored by atoms pumped into the upper laser level into photons, which are now inside the optical resonator. These proceed to bounce back and forth between the reflectors with a fraction $(1 - R)$ "escaping" from the resonator each time. This causes a decay of the pulse with a characteristic time constant (the "photon lifetime") given in (4.7-3) as

$$t_c \simeq \frac{nl}{c(1 - R)}$$

Both experiment and theory indicate that the total evolution of a giant laser pulse as described above is typically completed in $\sim 2 \times 10^{-8}$ second. We will consequently neglect the effect of population relaxation and pumping that take place during the pulse. We will also assume that the switching of the Q from the low to the high value is accomplished instantaneously.

The laser is characterized by the following variables: ϕ, the total number of photons in the optical resonator, $n \equiv (N_2 - N_1)V$; the total inversion; and t_c, the decay time constant for photons in the *passive* resonator. The exponential gain constant γ is proportional to n. The radiation intensity I thus grows with distance as $I(z)$

$= I_0 \exp(\gamma z)$ and $dI/dz = \gamma I$. An observer traveling with the wave velocity will see it grow at a rate

$$\frac{dI}{dt} = \frac{dI}{dz}\frac{dz}{dt} = \gamma\left(\frac{c}{n}\right) I$$

and thus the temporal exponential growth constant is $\gamma(c/n)$. If the laser rod is of length L while the resonator length is l, then only a fraction L/l of the photons is undergoing amplification at any one time and the average growth constant is $\gamma c(L/nl)$. We can thus write

$$\frac{d\phi}{dt} = \phi\left(\frac{\gamma c L}{nl} - \frac{1}{t_c}\right) \tag{6.9-1}$$

where $-\phi/t_c$ is the decrease in the number of resonator photons per unit time due to incidental resonator losses and to the output coupling. Defining a dimensionless time by $\tau = t/t_c$ we obtain, upon multiplying (6.9-1) by t_c,

$$\frac{d\phi}{d\tau} = \phi\left[\left(\frac{\gamma}{nl/cLt_c}\right) - 1\right] = \phi\left[\frac{\gamma}{\gamma_t} - 1\right]$$

where $\gamma_t = (nl/cLt_c)$ is the minimum value of the gain constant at which oscillation (that is, $d\phi/d\tau = 0$) can be sustained. Since, according to (5.3-3) γ is proportional to the inversion n, the last equation can also be written as

$$\frac{d\phi}{d\tau} = \phi\left[\frac{n}{n_t} - 1\right] \tag{6.9-2}$$

where $n_t = N_t V$ is the total inversion at threshold as given by (6.1-9).

The term $\phi(n/n_t)$ in (6.9-2) gives the number of photons generated by induced emission per unit of normalized time. Since each generated photon results from a single transition, it corresponds to a decrease of $\Delta n = -2$ in the total inversion. We can thus write directly

$$\frac{dn}{d\tau} = -2\phi\frac{n}{n_t} \tag{6.9-3}$$

The coupled pair of equations, (6.9-2) and (6.9-3), describes the evolution of ϕ and n. It can be solved easily by numerical techniques. Before we proceed to give the results of such calculation, we will consider some of the consequences that can be deduced analytically.

Dividing (6.9-2) by (6.9-3) results in

$$\frac{d\phi}{dn} = \frac{n_t}{2n} - \frac{1}{2}$$

and, by integration,

$$\phi - \phi_i = \frac{1}{2}\left[n_t \ln\frac{n}{n_i} - (n - n_i)\right]$$

Assuming that ϕ_i, the initial number of photons in the cavity, is negligible, we obtain

$$\phi = \frac{1}{2}\left[n_t \ln \frac{n}{n_i} - (n - n_i)\right] \tag{6.9-4}$$

for the relation between the number of photons ϕ and the inversion n at any moment. At $t \gg t_c$ the photon density ϕ will be zero so that setting $\phi = 0$ in (6.9-4) results in the following expression for the final inversion n_f:

$$\frac{n_f}{n_i} = \exp\left[\frac{n_f - n_i}{n_t}\right] \tag{6.9-5}$$

This equation is of the form $(x/a) = \exp(x - a)$, where $x = n_f/n_t$ and $a = n_i/n_t$, so that it can be solved graphically (or numerically) for n_f/n_i as a function of n_i/n_t.[15] The result is shown in Figure 6-27. We notice that the fraction of the energy originally stored in the inversion that is converted into laser oscillation energy is $(n_i - n_f)/n_i$ and that it tends to unity as n_i/n_t increases.

[15]This can be done by assuming a value of a and finding the corresponding x at which the plots of x/a and $\exp(x - a)$ intersect.

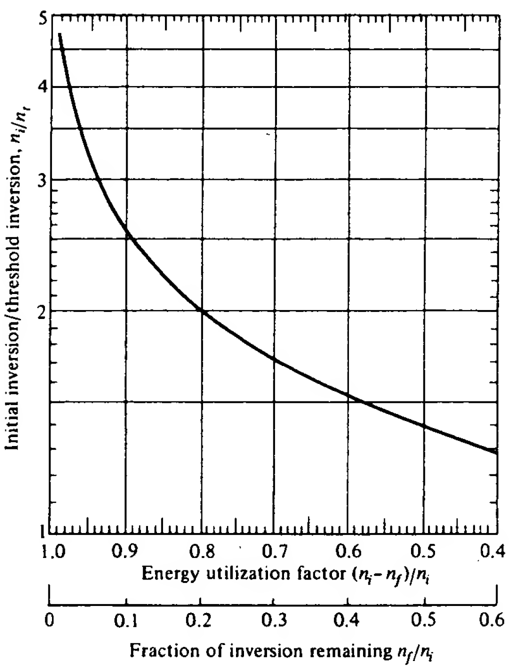


Figure 6-27 Energy utilization factor $(n_i - n_f)/n_i$ and inversion remaining after the giant pulse. (After Reference [19].)

The instantaneous power output of the laser is given by $P = \phi h\nu/t_c$, or, using (6.9-4), by

$$P = \frac{h\nu}{2t_c}\left[n_t \ln \frac{n}{n_i} - (n - n_i)\right] \tag{6.9-6}$$

Of special interest to us is the peak power output. Setting $\partial P/\partial n = 0$ we find that maximum power occurs when $n = n_t$. Putting $n = n_t$ in (6.9-6) gives

$$P_p = \frac{h\nu}{2t_c}\left[n_t \ln \frac{n_t}{n_i} - (n_t - n_i)\right] \tag{6.9-7}$$

for the peak power. If the initial inversion is well in excess of the (high-Q) threshold value (that is, $n_i \gg n_t$), we obtain from (6.9-7)

$$(P_p)_{n_i \gg n_t} \simeq \frac{n_i h\nu}{2t_c} \tag{6.9-8}$$

Since the power P at any moment is related to the number of photons ϕ by $P = \phi h\nu/t_c$, it follows from (6.9-8) that the maximum number of stored photons inside the resonator is $n_i/2$. This can be explained by the fact that if $n_i \gg n_t$, the buildup of the pulse to its peak value occurs in a time short compared to t_c so that at the peak of the pulse, when $n = n_t$, most of the photons that were generated by stimulated emission are still present in the resonator. Moreover, since $n_i \gg n_t$, the number of these photons $(n_i - n_t)/2$ is very nearly $n_i/2$.

A typical numerical solution of (6.9-2) and (6.9-3) is given in Figure 6-28.

To initiate the pulse we need, according to (6.9-2) and (6.9-3), to have $\phi_i \neq 0$. Otherwise the solution is trivial ($\phi = 0$, $n = n_i$). The appropriate value of ϕ_i is

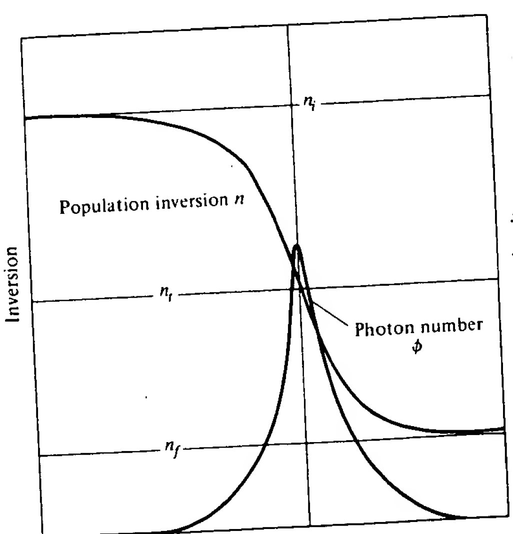


Figure 6-28 Inversion and photon density during a giant pulse. (After Reference [19].)

usually estimated on the basis of the number of spontaneously emitted photons within the acceptance solid angle of the laser mode at $t = 0$. We also notice, as discussed above, that the photon density, hence the power, reaches a peak when $n = n_t$. The energy stored in the cavity ($\propto \phi$) at this point is maximum, so stimulated transitions from the upper to the lower laser levels continue to reduce the inversion to a final value $n_f < n_t$.

Numerical solutions of (6.9-2) and (6.9-3) corresponding to different initial inversions n_i/n_t are shown in Figure 6-29. We notice that for $n_i \gg n_t$ the rise time becomes short compared to t_c but the fall time approaches a value nearly equal to

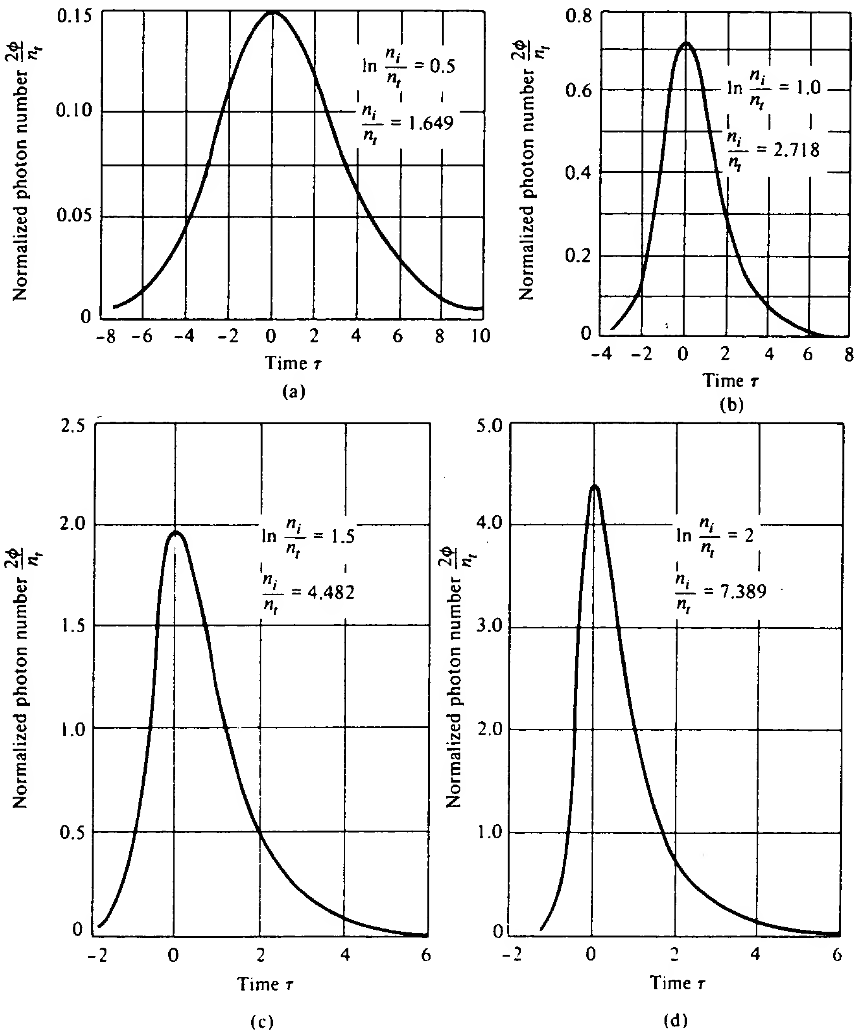


Figure 6-29 Photon number vs. time in central region of giant pulse. Time is measured in units of photon lifetime. (After Reference [19].)

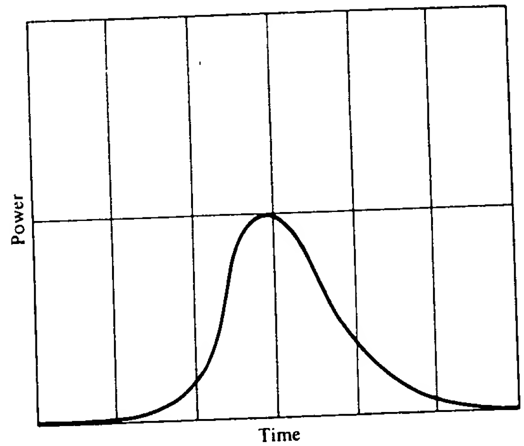


Figure 6-30 An oscilloscope trace of the intensity of a giant pulse. Time scale is 20 ns per division.

t_c. The reason is that the process of stimulated emission is essentially over at the peak of the pulse ($\tau = 0$) and the observed output is due to the free decay of the photons in the resonator.

In Figure 6-30 we show an actual oscilloscope trace of a giant pulse. Giant laser pulses are used extensively in applications that require high peak powers and short duration. These applications include experiments in nonlinear optics, ranging, material machining and drilling, initiation of chemical reactions, and plasma diagnostics.

Numerical Example: Giant Pulse Ruby Laser

Consider the case of pink ruby with a chromium ion density of $N = 1.58 \times 10^{19}$ cm^{-3}. Its absorption coefficient is taken from Figure 7-4, where it corresponds to that of the R_1 line at 6943 Å, and is $\alpha \simeq 0.2$ cm^{-1} (at 300 K). Other assumed characteristics are:

$$l = \text{length of ruby rod} = 10 \text{ cm}$$

$$A = \text{cross-sectional area of mode} = 1 \text{ cm}^2$$

$$(1 - R) = \text{fractional intensity loss per pass} = 20 \text{ percent}$$

$$n = 1.78$$

Since, according to (5.3-3), the exponential loss coefficient is proportional to $N_1 - N_2$, we have

$$\alpha(\mathrm{cm}^{-1}) = 0.2 \frac{N_1 - N_2}{1.58 \times 10^{19}} \tag{6.9-9}$$

Thus, at room temperature, where $N_2 \ll N_1$ when $N_1 - N_2 \cong 1.58 \times 10^{19}$ cm^{-3}, and (6.9-9) yields $\alpha = 0.2$ cm^{-1} as observed. The expression for the gain coefficient follows directly from (6.9-9):

$$\gamma(\mathrm{cm}^{-1}) = 0.2 \frac{N_2 - N_1}{1.58 \times 10^{19}} = 0.2 \frac{n}{1.58 \times 10^{19}\, V} \tag{6.9-10}$$

where n is the total inversion and $V = AL$ is the crystal volume in cm^3.

Threshold is achieved when the net gain per pass is unity. This happens when

$$e^{\gamma_t l} R = 1 \quad \text{or} \quad \gamma_t = -\frac{1}{l} \ln R \tag{6.9-11}$$

where the subscript t indicates the threshold condition.

Using (6.9-10) in the threshold condition (6.9-11) plus the appropriate data from above gives

$$n_t = 1.8 \times 10^{19} \tag{6.9-12}$$

Assuming that the initial inversion is $n_i = 5n_t = 9 \times 10^{19}$, we find from (6.9-8) that the peak power is approximately

$$P_p = \frac{n_i h\nu}{2t_c} = 5.1 \times 10^9 \text{ watts} \tag{6.9-13}$$

where $t_c = nl/c(1 - R) \simeq 2.5 \times 10^{-9}$ s.

The total pulse energy is

$$\mathscr{E} \sim \frac{n_i h\nu}{2} \sim 13 \text{ joules}$$

while the pulse duration (see Figure 6-29) $\simeq 3t_c \simeq 7.5 \times 10^{-9}$ s.

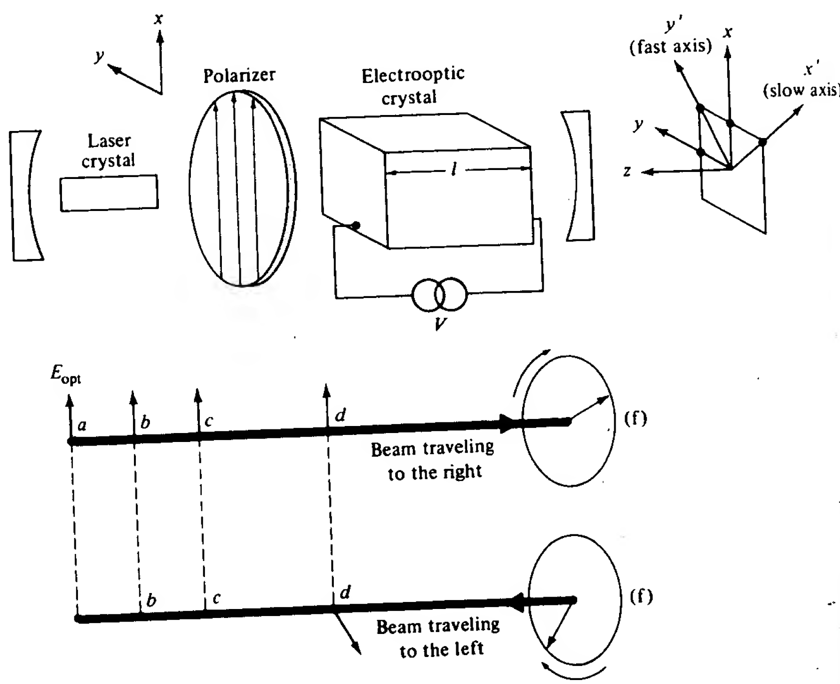
Methods of Q-Switching

Some of the schemes used in Q-switching are:

1. Mounting one of the two end reflectors on a rotating shaft so that the optical losses are extremely high except for the brief interval in each rotation cycle in which the mirrors are nearly parallel.
2. The inclusion of a saturable absorber (bleachable dye) in the optical resonator, see References [13–15]. The absorber whose opacity decreases (saturates) with increasing optical intensity prevents rapid inversion depletion due to buildup of oscillation by presenting a high loss to the early stages of oscillation during which the slowly increasing intensity is not high enough to saturate the absorp-

tion. As the intensity increases the loss decreases, and the effect is similar, but not as abrupt, as that of a sudden increase of Q.

3. The use of an electrooptic crystal (or liquid Kerr cell) as a voltage-controlled gate inside the optical resonator. It provides a more precise control over the losses (Q) than schemes 1 and 2. Its operation is illustrated by Figure 6-31 and is discussed in some detail in the following. The control of the phase delay in



For beam traveling to right:

At point d,

$$E_{x'} = \frac{E}{\sqrt{2}} \cos \omega t, \qquad E_{y'} = \frac{E}{\sqrt{2}} \cos \omega t$$

The optical field is linearly polarized with its electric field vector parallel to x

At point f,

$$E_{x'} = \frac{E}{\sqrt{2}} \cos\left(\omega t + kl + \frac{\pi}{2}\right), \qquad E_{y'} = \frac{E}{\sqrt{2}} \cos(\omega t + kl)$$

Circularly polarized

For beam traveling to left:

At point f,

$$E_{x'} = -\frac{E}{\sqrt{2}} \cos\left(\omega t + kl + \frac{\pi}{2}\right), \qquad E_{y'} = -\frac{E}{\sqrt{2}} \cos(\omega t + kl)$$

Circularly polarized

At point d,

$$E_{x'} = -\frac{E}{\sqrt{2}} \cos(\omega t + 2kl + \pi), \qquad E_{y'} = -\frac{E}{\sqrt{2}} \cos(\omega t + 2kl)$$

Linearly polarized along y'

Figure 6-31 Electrooptic crystal used as voltage-controlled gate in Q-switching a laser.

the electrooptic crystal by the applied voltage is discussed in detail in Chapter 9.

During the pumping of the laser by the light from a flashlamp, a voltage is applied to the electrooptic crystal of such magnitude as to introduce a $\pi/2$ relative phase shift (retardation) between the two mutually orthogonal components (x' and y') that make up the linearly polarized (x) laser field. On exiting from the electrooptic crystal at point f, the light traveling to the right is circularly polarized. After reflection from the right mirror, the light passes once more through the crystal. The additional retardation of $\pi/2$ adds to the earlier one to give a total retardation of π, thus causing the emerging beam at d to be linearly polarized along y and consequently to be blocked by the polarizer.

It follows that with the voltage on, the losses are high, so oscillation is prevented. The Q-switching is timed to coincide with the point at which the inversion reaches its peak and is achieved by a removal of the voltage applied to the electrooptic crystal. This reduces the retardation to zero so that state of polarization of the wave passing through the crystal is unaffected and the Q regains its high value associated with the ordinary losses of the system.

6.10 HOLE-BURNING AND THE LAMB DIP IN DOPPLER-BROADENED GAS LASERS

In this section we concern ourselves with some of the consequences of Doppler broadening in low-pressure gas lasers.

Consider an atom with a transition frequency $\nu_0 = (E_2 - E_1)/h$ where 2 and 1 refer to the upper and lower laser levels, respectively. Let the component of the velocity of the atom parallel to the wave propagation direction be v. This component, thus, has the value

$$v = \frac{\mathbf{v}_{\text{atom}} \cdot \mathbf{k}}{k} \tag{6.10-1}$$

where the electromagnetic wave is described by

$$\mathbf{E} = \mathbf{E}e^{i(2\pi\nu t - \mathbf{k}\cdot\mathbf{r})} \tag{6.10-2}$$

An atom moving with a constant velocity $\mathbf{v}$, so that $\mathbf{r} = \mathbf{v}t + \mathbf{r}_0$, will experience a field

$$\begin{aligned}\mathbf{E}_{\text{atom}} &= \mathbf{E}e^{i[2\pi\nu t - \mathbf{k}\cdot(\mathbf{r}_0 + \mathbf{v}t)]}\\ &= \mathbf{E}e^{i[(2\pi\nu - \mathbf{v}\cdot\mathbf{k})t - \mathbf{k}\cdot\mathbf{r}_0]}\end{aligned} \tag{6.10-3}$$

and will thus "see" a Doppler-shifted frequency

$$\nu_D = \nu - \frac{\mathbf{v}\cdot\mathbf{k}}{2\pi} = \nu - \frac{v}{c}\,\nu \tag{6.10-4}$$

where in the second equality we took $n = 1$ so that $k = 2\pi\nu/c$ and used (6.10-1).

The condition for the maximum strength of interaction (that is, emission or